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Binary evolution and neutron stars in globular clusters

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Abstract. Improved observations of globular clusters are uncovering a large number of radio pulsars and of X-ray sources. The latter include binaries in which a neutron star or a white dwarf accretes matter from a companion, recycled pulsars, and magnetically active binaries. Most of these sources originate from close encounters between stars in the cluster core; magnetically active binaries and some cataclysmic variables have evolved from primordial binaries. The formation rate through close encounters scales differently with the central density and core radius of the cluster than the probability for a single binary to be perturbed by an encounter. This is exploited in some preliminary observational tests of the close encounter hypothesis.
Accreting black holes have been found in globular clusters with other galaxies; the absence of such holes in the Milky Way clusters is compatible with the small number expected.

1. Introduction

Interest in binaries in globular clusters revived when the first X-ray maps of the whole sky showed an overabundance in globular clusters of bright ($L_x \gtrsim 10^{36}$ erg/s) sources: whereas globular clusters contain $\sim$0.1% of the stars of our galaxy, they contain about 10% of the bright X-ray sources. It was soon suggested that close stellar encounters in globular cluster cores were responsible (Clark 1975). With more sensitive instruments faint ($L_x \lesssim 10^{35}$ erg/s) sources were detected in globular clusters: 8 in 8 clusters with the Einstein satellite in the 1980s (Hertz & Grindlay 1983), and 57 in 23 clusters with ROSAT in the 1990s (Verbunt 2001). Of the latter 57, 17 are more than two core radii from the cluster core. In the new millenium, Chandra detected about 100 faint X-ray sources in 47 Tuc alone, and 25-40 sources in each of NGC6752, NGC6397, and NGC6440 (Grindlay et al. 2001a,b, Pooley et al. 2002a,b; Fig. 1). First XMM results are now being published (Webb et al. 2002; Webb, Gendre in these proceedings).

Radio pulsars have also been detected in large numbers: 20 in 47 Tuc alone, 8 in M15, 5 in NGC6752, and some 20 in a dozen other clusters (Freire et al. 2001, D'Amico et al. 2002, and lists in e.g. Phinney 1992, Lyne 1994, and on the Web: Freire 2002). Almost all of these are recycled radio pulsars, i.e. have obtained their rapid rotation (and probably also their low magnetic field) through accretion of matter and angular momentum from a companion star.

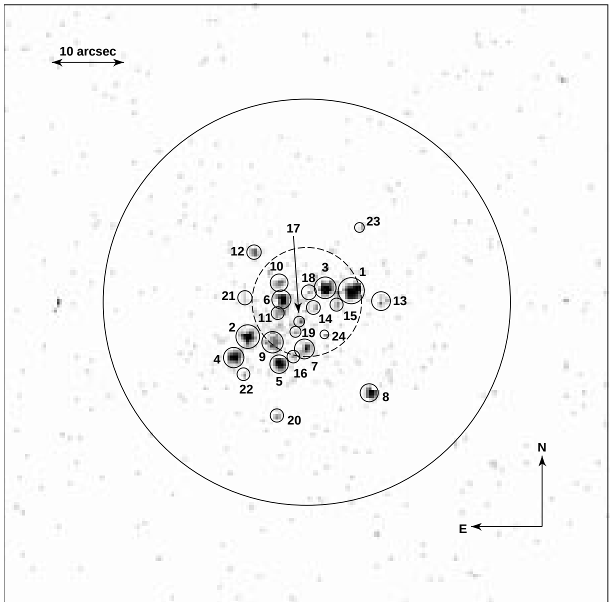

Figure 1. X-ray sources in NGC6440. Sources 1 certainly and 2, 3 and 5 probably are neutron stars accreting from a companion star at a low rate; most of the other sources are probably white dwarfs accreting from a companion, i.e. cataclysmic variables. Note that the source distribution extends beyond the core radius (indicated with a dashed circle), but remains well within the half-mass radius (solid-line circle). From Pooley et al. (2002b).

At the moment 13 bright X-ray sources are known in a total of 12 globular clusters. The only binaries producing such luminosities are low-mass X-ray binaries, in which a neutron star or black hole accretes mass from a companion. Because of the occurrence of X-ray bursts, 12 of the 13 bright sources in clusters are known to be neutron stars; the 13th one is – on the basis of its X-ray spectrum – probably a neutron star as well (in 't Zand et al. 1999, White & Angelini 2001). Thus there is no evidence for accreting black holes in any globular cluster with our galaxy. The dim sources with $10^{32.5} \lesssim L_x(\mathrm{erg/s}) \lesssim 10^{35}$ are most likely neutron stars accreting at a low rate; most of the dimmer sources ($\lesssim 10^{32.5}$) are probably cataclysmic variables, but some of them are recycled radio pulsars, or magnetically active close binaries ('RS CVns') – see e.g. Fig.8 in Verbunt et al. (1997); and the articles by Cool and Edmonds in these proceedings.

In this review I discuss the formation and evolution of these X-ray sources and recycled radio pulsars. Formation through evolution from a primordial binary is compared with formation via close stellar encounters in Section 2. The evolution of binaries with a compact star (neutron star or white dwarf) is described in Section 3. Various tests of our theoretical ideas are made by comparison with observation in Section 4. A brief discussion in Section 5 of X-ray sources in globular clusters of other galaxies, precedes the Summary.

2. Formation: evolution versus encounters

All binaries that we observe in the disk of the galaxy have evolved from primordial binaries. Binaries which are very common in the disk, apparently are commonly formed via ordinary binary evolution. If we find such binaries in a globular cluster, it is likely that they have evolved from primordial binaries as well. Examples of such binaries are RS CVn's and contact binaries. On the other hand, binaries with neutron stars (or black holes) are extremely rare in the disk; such a binary in a globular cluster is formed almost certainly from a close encounter of a neutron star with a single star or with a binary. Cataclysmic variables are in between these extremes: in globular clusters with dense cores, such as 47 Tuc, most may have formed via stellar encounters; but in more open clusters, as ω Cen, they may have evolved from primordial binaries (Verbunt & Meylan 1988).

We consider tidal capture first. The encounter rate is proportional to the encounter cross section A, to the relative velocity v, and to the numbers n_c and n per unit volume, of compact stars and ordinary stars respectively. To obtain the encounter rate Γ for the cluster as a whole, one integrates over the cluster volume; because of the high densities in the core this roughly corresponds to multiplying the central rate with the core volume. Due to gravitational focussing, the cross section A of close encounters is proportional to the radius of the star R and inversely proportional to the square of the velocities v (see also Davies, these proceedings). Thus (Hut & Verbunt 1983):

$$\Gamma \propto \int n_c n A v dV \propto \int \frac{n_c n R}{v} dV \propto \frac{\rho_o^2 r_c^3}{v} R \tag{1}$$

Where ρ_o is the central mass density and r_c the core radius. An analogous reasoning gives the exchange encounter rate

$$\Gamma_e \propto \int n_c n_b A_b v dV \propto \int \frac{n_c n_b a}{v} dV \propto \frac{\rho_o^2 r_c^3}{v} a \tag{2}$$

where n_b is the number of binaries per unit volume, and a the semi-major axis of the binary. The ratio of tidal capture to exchange encounters is roughly

$$\frac{\Gamma}{\Gamma_e} \sim \frac{R}{a} \frac{n}{n_b} \tag{3}$$

The susceptibility of large binaries to close encounters with other cluster stars implies that many such binaries are dissolved by passing stars. As a result, the formation of cataclysmic variables from evolution of primordial binaries, which passes through a stage in which the binary is very wide, is suppressed in globular clusters (Davies 1997).

The importance of tidal capture is under debate, because it leads to an initially highly eccentric orbit: the circularization of this orbit is accompanied by dissipation of an amount of energy ΔE which is comparable to the binding energy E_b of the ordinary star:

$$\frac{\Delta E}{E_b} \simeq \left(\frac{-GMm}{2a_c} \right) / \left(\frac{3GM^2}{5R} \right) \simeq \frac{5}{6} \frac{m}{M} \frac{R}{a_c} \tag{4}$$

where M and R are the mass and radius of the tidally distorted star, m the mass of the other star and a_c the radius of the circularized orbit. If the energy is dissipated more rapidly than it can be radiated or convected away, the ordinary star is destroyed, and no binary remains (Ray et al. 1987).

Tidal capture, if successful, tends to lead to orbits with $a \simeq 3R$, i.e. to orbits with short periods, less than 1 day, say. In contrast, exchange encounters favour wide orbits, and in addition will cause a recoil velocity of the binary which may take it out of the cluster core – perhaps even out of the cluster! Tidal capture can also occur during an encounter of a single star with a binary, when the three-body intraction brings two stars close to one another.

A much debated point of uncertainty is the number of neutron stars that remain in a cluster after their formation: young pulsars obtain a kick velocity at birth, as witnessed by the observed velocities of radio pulsars. It has been argued that these velocities are higher than the escape velocities of globular clusters, implying that no neutron stars would be retained. In my opinion, the velocities of pulsars have been rather overestimated, due to underestimates of the errors in proper motions and distances (see Hartman 1997). A recent list of 16 accurate velocities obtained with VLBI (Brisken et al. 2002) indicates that as much as a third of neutron stars is born with velocities less than $\sim 50\,\mathrm{km/s}$. Thus as many as 30% of the neutron stars born in globular clusters are retained. Loss of neutron stars from globular clusters is counteracted by mass segregation, which concentrates the (relatively heavy) neutron stars in the cores, where the encounter rates are highest (Verbunt & Meylan 1988).

The presence of pulsars in globular clusters is interesting, as they constrain the amount of non-luminous matter (to which they themselves do not contribute significantly, as the mass in white dwarfs is always much higher). A pulsar which has a radial velocity v_r with respect to us, will be observed at a period shifted by $\Delta P = (v_r/c)P$. If the pulsar is accelerated, a period derivative $\Delta\dot{P} = (a_r/c)P$ is measured on top of the intrinsic period derivative. Since the latter is always positive, measurement of a negative period derivative indicates that acceleration dominates, and thus provides an estimate of the mass density. Phinney (1992) has shown that the pulsars in M15 put a tight constraint on the mass of any black hole in the cluster core – the most likely mass of the cluster core can in fact be explained by the sum of visible stars and white dwarfs, if the mass distribution initially followed the Salpeter function, i.e. no black hole is required.

3. Binary evolution

The evolution of a binary with a neutron star (or black hole, or white dwarf) is a complex topic, which is described only briefly here. For more detail, see e.g. Verbunt (1993). The mass transfer in a close binary in which a companion to a neutron star fills its Roche lobe, can be driven by loss of angular momentum from the orbit. If a compact star of mass m receives matter from a Roche-filling star with mass M, the mass transfer rate is roughly given by $-\dot{M}/M \simeq \dot{m}/m \sim \dot{J}/J_b$, where J_b is the binary angular momentum. For a main sequence star donor, the orbital period P_b is roughly given by $P_b(\mathrm{hr}) \simeq 8M(M_\odot)$, and gravitational radiation leads to a mass transfer rate of order $10^{-10}M_\odot\,\mathrm{yr}^{-1}$. When accreted onto a neutron star this leads to an X-ray luminosity $\sim 10^{36}$ $\mathrm{erg\,s}^{-1}$. The observation of higher X-ray luminosities in such binaries have lead to the suggestion of additional mechanisms of loss of angular momentum, such as 'magnetic braking', in which a stellar wind carries away angular momentum efficiently because it is tied to the stellar surface out to large radii by magnetic field lines. The efficiency of magnetic braking is unknown; recent studies of stellar rotation in young star clusters suggests that its importance may have been over-estimated (e.g. Tinker et al. 2002). The mass transfer rate may also be affected if the donor is irradiated by X-rays from the accreting star. If this irradiation causes the star to expand, it may enhance the mass transfer rate; if it causes an enhanced stellar wind and mass loss from the binary, it may reduce the mass transfer rate. Whether irradition is important is not known; the absence of strong heating effects in most well-observed low-mass X-ray binaries suggests that the accretion disk shield the secondary from X-rays emitted near the accreting star.

If the mass transfer stops, the neutron star may spin rapidly enough to switch on as a radio pulsar. The standard description of evolution through loss of angular momentum does not foresee such an end however, but rather predicts an everlasting, albeit ever lower, mass transfer. The observation of close binaries with a pulsar shows that the mass transfer does, in fact, stop. Perhaps if the mass transfer is variable, it may reach a state sufficiently low that the pulsar switches on, after which the pulsar wind itself can prevent further mass transfer. Once more, the details of such a process are not understood.

The star that donates matter to a neutron star can also be a white dwarf; in that case the orbital period is roughly $P_b(\mathrm{s}) \simeq 50/M(M_\odot)$. In such a binary, stable mass tranfer can only occur if the white dwarf has a mass less than $\sim 0.66 M_\odot$; a more massive white dwarf will expand so rapidly with mass loss that it is disrupted within one binary revolution (Verbunt & Rappaport 1988).

If the initial binary is too wide, loss of angular momentum is not important, and mass transfer will only start when the donor expands as it ascends the (sub)giant branch. In that case $-\dot{M}/M \simeq \dot{m}/m \sim \dot{R}/R$, where R is the stellar radius. According to stellar evolution, $\dot{R}$ increases with R, and thus the mass transfer rate is expected to be higher in wide binaries. Conservation of angular momentum widens the orbit as mass is transferred from the (sub)giant to the neutron star; typically, the orbital period has increased by a factor ~ 7 by the time that the whole envelope has been transferred. From that point on, the binary will consist of a neutron star and a white dwarf. The mass transfer puts an early end to the growth of the giant core, and the emerging white dwarf is undermassive; typically $M_{wd} \sim 0.25 M_\odot$. Tidal interaction during the mass transfer leads to an orbit which is almost but not quite circular. The observed white dwarf masses and the relation between orbital period and (small) eccentricity provide strong support for this evolution scenario for recycled pulsars with white dwarf companions (Phinney 1992).

It is observed that rather more millisecond pulsars are found in globular clusters than can be formed from the currently observed X-ray binaries. A solution could be that most recycled pulsars were spun up in binaries with donors of intermediate mass, 1-3 $M_\odot$ (Davies & Hansen 1998).

4. Formation through close encounters: testing the theory

With a growing number of binaries with neutron stars observed in globular clusters, we can start comparing their observed properties with the theory of formation. To do so, we refer to Eqs. 1 and 2, and note that the virial theorem relates the velocity dispersion to the central density and core radius of the

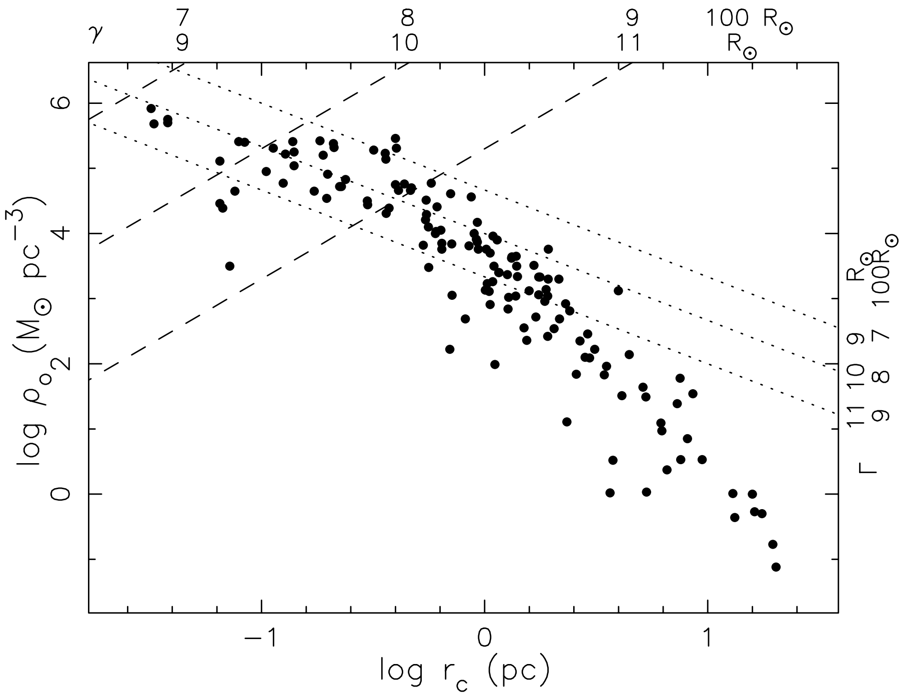

Figure 2. Central density as a function of core radius for globular clusters in our Galaxy. Data from Harris (1996, revision of June 22 1999). Dotted lines indicate loci of constant formation rate Γ in the cluster, according to Eq. 6; approximate numerical factors are indicated on the right for $R = R_\odot$ (applicable to tidal capture) and for $a = 100R_\odot$, $n_b \simeq n$ (exchange encounters) where x indicates one encounter per 10^x yr. Dashed lines indicate lines of constant encounter rate γ for *one single* system, according to Eq. 7; approximate numerical factors are indicated on top.

cluster:

$$v \propto \sqrt{\rho_o}\, r_c \tag{5}$$

Therefore

$$\Gamma \propto {\rho_o}^{1.5}{r_c}^2 R \qquad \text{and} \qquad \Gamma_e \propto {\rho_o}^{1.5}{r_c}^2 a \tag{6}$$

In Figure 2 we plot lines of constant Γ in a graph showing the central density as function of core radius for the globular clusters of our Milky Way. The numbers indicated use the King model for the numerical factor in Eq. 5. The formation rate of binaries with neutron stars can be high because the central density is high, or because the cluster has a large core.

The situation is different for the probability that a binary, once formed, will undergo a subsequent encounter which may change or destroy it. The rate at which a binary undergoes close encounters is given by

$$\gamma = nAv \propto \frac{\rho_o}{v}\, a \propto \frac{{\rho_o}^{0.5}}{r_c}\, a \tag{7}$$

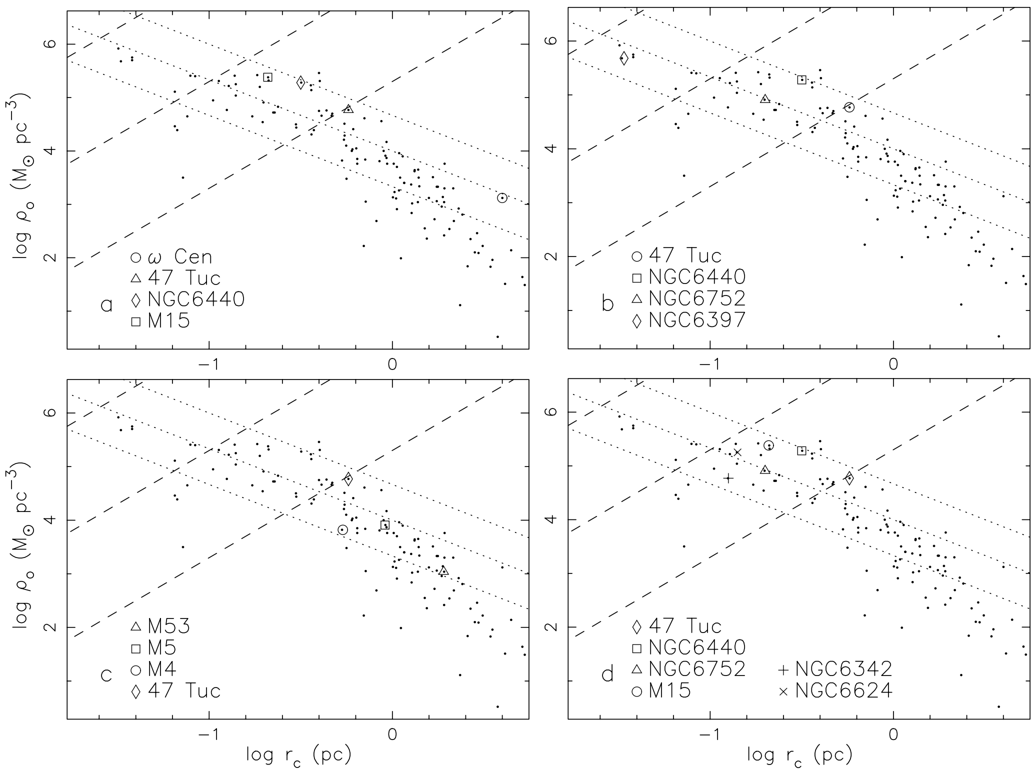

Figure 3. Various – preliminary – tests on neutron stars in binaries in globular clusters. a) Number of binaries with neutron stars, b) Slope of the luminosity function of the X-ray sources, c) Orbital periods of pulsar binaries, d) Pulsar pulse periods. For details see text.

Lines of constant γ are also shown in Figure 2. We see that in the clusters with the highest central density, both close binaries (formed from tidal capture) and wider binaries (formed from an exchange encounter) are affected by subsequent encounters.

The exact encounter rates in a cluster depend on the mass function in the core, hence on the mass segregation, on the fraction of stars in binaries, and on the period distribution of the binaries. For encounters involving neutron stars, the rates depend on the retention fraction of neutron stars. Also, it is not clear how one should treat a collapsed cluster. All these factors are different in different clusters. In the absence of detailed information on most clusters, we can only perform preliminary tests, in which we ignore these factors.

One such test is that the probability that a cluster contains a bright X-ray source scales with Γ, i.e. with the collision number $\Sigma \equiv \rho_c^2 r_c^3/v$. The bright X-ray sources pass this test (Verbunt & Hut 1987). Johnston et al. (1992) claim to find that the probability that a cluster contains a recycled pulsar is less dependent on ρ_c, viz. $\propto \rho_c^{1.5}$; however, this lower dependence may be an artefact of their assumption that v is the same in all clusters (compare eqs.1 and 6).

Some further tests are illustrated in Figure 3. Numbers used in these tests for the X-ray sources / radio pulsars were taken from: ω Cen Rutledge et al. 2002 / –; 47 Tuc Grindlay et al. 2001a / Freire et al. 2001; NGC6440 Pooley et al. 2002b / Lyne et al. 1996; M15 White & Angelini 2001 / Phinney 1992; NGC6752 Pooley et al. 2001a / D'Amico et al. 2002; NGC6397 Grindlay et al. 2002b / –. Numbers for other pulsars from: M53 Kulkarni et al. 1991; M5 Anderson et al. 1997; M4 Thorsett et al. 1999; NGC6342 Lyne et al. 1993; NGC6624 Biggs et al. 1994.

a) ω Cen contains one neutron star X-ray binary and no (known) pulsar, whereas 47 Tuc, NGC6440 and M 15 contain respectively 2, 4 and 2 neutron star X-ray binaries and 20, 1 and 8 (known) pulsars. ω Cen indeed has a lower value of Γ than the other three clusters.

b) The slope p of the X-ray luminosity function $dN(L_\mathrm{x}) \propto {L_\mathrm{x}}^{-p} d\ln L_\mathrm{x}$ (including neutron star binaries, cataclysmic variables, pulsars, and magnetically active binaries) is steep in 47 Tuc ($p = 0.8$), intermediate in NGC6752 and NGC6440 ($p = 0.5$), and shallow in NGC6397 ($p = 0.3$). This may be related to the high value of γ in NGC6397, which prevents a binary from evolving without being interfered with; thus ordinary binaries in this cluster have largely been destroyed (Pooley et al. 2002b).

c) The clusters M4 and M53 contain pulsars in binaries with long orbital periods of 191 and 256 days; these clusters indeed have a low value of γ, i.e. wide binaries are not affected by encounters. 47 Tuc, with a higher value for γ, does not contain pulsars in binaries with periods longer than 2.3 d. (The binary in M4 in fact has a companion third star at about $6000\,R_\odot$; the outer binary has only an expected life time of 10^8 yr.)

d) It has been an outstanding puzzle that all pulsars in 47 Tuc and NGC6752 have very short pulse periods (< 7.6 ms and < 9.0 ms), whereas those in M15 have pulse periods ranging up to 111 ms. Long periods also are found in NGC6342, NGC6440 and NGC6624 (1.0 s, 289 ms and 379 ms). There is no obvious correlation of the pulse period range with Γ or γ.

The encounter hypothesis for the formation of binaries with a neutron star survives the preliminary tests.

5. Globular clusters in other galaxies

With the Einstein and Rosat satellites, X-ray sources in globular clusters of M31 have been discovered. On the whole it appears that the properties of these sources are not significantly different from those in our galaxy (Supper et al. 1997). Thanks to Chandra it has become possible to observe X-ray sources in other galaxies, with fairly accurate positions. As a result, it has been found that globular clusters in other galaxies contain X-ray sources with luminosities above 10^{39} erg/s (Angelini et al. 2001, Sarazin et al. 2001, Kundu et al. 2002) At such luminosities, the X-ray sources are probably black holes accreting from a binary companion; this is confirmed by the soft X-ray spectrum when it can be measured. Apparently the clusters in those galaxies *do* contain black holes in binaries. This is not necessarily a significant difference with the clusters in our galaxy: if 20 % of the bright X-ray sources in globular clusters would contain black holes, the expectation value for such binaries in our globular cluster system would be 2 or 3. For this expectation value the probability of finding zero is appreciable.

A first contribution by XMM is the detection of a faint X-ray source in Mayall II, the largest cluster of M31 (Verbunt, Meylan & Mendez, in preparation).

6. Summary

The overabundance in globular clusters of binaries with neutron stars is due to formation of such binaries in close encounters. When the neutron star accretes matter from its companion, it is an X-ray source. Chandra observations show that there are 5 to 10 times as many neutron stars accreting at low rates than at high rates. Whether this is related to their formation mechanism is not clear, since a similar overabundance is found in the galactic disk where binaries with neutron stars evolve from primordial binaries (Cornelisse et al. 2002). When the neutron star stops accreting, it can become a radio pulsar. Most cataclysmic variables in globular clusters are also formed in close encounters.

Tidal capture leads to a short orbital period, and has no associated recoil velocity; exchange encounters favour wide binaries and do involve recoil. Most neutron star binaries are found in or close to the cores of globular clusters. Most binaries with a pulsar have relatively short orbital periods, and mass functions that indicate a mass of $\simeq 0.25 M_{\odot}$ for their white dwarf companion. In my view, this indicates that these binaries were formed via tidal capture. Some binaries have rather longer orbital periods, and some are far from the cluster core. These binaries were probably formed via exchange encounters. From the X-ray luminosity function it appears that magnetically active binaries – that make up the low–luminosity end of the distribution – are destroyed in dense clusters.

Observations of globular clusters in other galaxies show that black holes are present in them. It would be interesting to investigate whether any of the low–luminosity X-ray binaries in the globular clusters of our galaxy contains a black hole. The small number of X-ray sources in clusters of our Galaxy hampers comparison of their properties with those in globular clusters systems of other galaxies.

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